ACME III EDM blinding

“Inconsistent” blinding
(For asymmetry)

Two “natural” ways to blind

Blind in parity basis (ACME II and I)

$$\omega^{\mathcal{N}\mathcal{E}}_{\text{blinded}} = \omega^{\mathcal{N}\mathcal{E}} + \omega_{\text{blind}}$$
$$\Phi^{\mathcal{N}\mathcal{E}}_{\text{blinded}} = \Phi^{\mathcal{N}\mathcal{E}} + \omega_{\text{blind}}\ \tau_j$$
$$\mathcal{A}^{\mathcal{N}\mathcal{E}}_{\text{blinded}} = \mathcal{A}^{\mathcal{N}\mathcal{E}} + 2\mathcal{C}\omega_{\text{blind}}\ \tau_j$$
$$\mathcal{A}^{p\neq\mathcal{N}\mathcal{E}}_{\text{blinded}} = \mathcal{A}^{p\neq\mathcal{N}\mathcal{E}}$$

Blind in state basis

$$\omega_{\text{blinded, state}} = \omega_{state} + \omega_{\text{blind}}\tilde{\mathcal{N}}\tilde{\mathcal{E}}$$
$$\Phi_{\text{blinded, state}} = \Phi_{state} + \omega_{\text{blind}}\tau_j\tilde{\mathcal{N}}\tilde{\mathcal{E}}$$
$$\mathcal{A}_{\text{blinded, state}} = \mathcal{A}_{state} + 2\mathcal{C}_{\text{state}}\omega_{\text{blind}}\tau_j\tilde{\mathcal{N}}\tilde{\mathcal{E}}$$

Blind in state basis, when viewed in parity basis

$$\omega^{\mathcal{N}\mathcal{E}}_{\text{blinded}} = \omega^{\mathcal{N}\mathcal{E}} + \omega_{\text{blind}}$$
$$\Phi^{\mathcal{N}\mathcal{E}}_{\text{blinded}} = \Phi^{\mathcal{N}\mathcal{E}} + \omega_{\text{blind}}\ \tau_j$$
$$\mathcal{A}^{\mathcal{N}\mathcal{E}}_{\text{blinded}} = \mathcal{A}^{\mathcal{N}\mathcal{E}} + 2\omega_{\text{blind}}\ \tau_j f^{\mathcal{N}\mathcal{E}}(\mathcal{C}^{\text{nr}}, \mathcal{C}^{\mathcal{N}}, \mathcal{C}^{\mathcal{E}}, \mathcal{C}^{\mathcal{B}}, \mathcal{C}^{\mathcal{N}\mathcal{E}}, \cdots)$$
$$\mathcal{A}^{p\neq\mathcal{N}\mathcal{E}}_{\text{blinded}} = \mathcal{A}^{p\neq\mathcal{N}\mathcal{E}} + 2\omega_{\text{blind}}\ \tau_j f^{p\neq\mathcal{N}\mathcal{E}}(\mathcal{C}^{\text{nr}}, \mathcal{C}^{\mathcal{N}}, \mathcal{C}^{\mathcal{E}}, \mathcal{C}^{\mathcal{B}}, \mathcal{C}^{\mathcal{N}\mathcal{E}}, \cdots),$$

where f is some function that depends on the values of the contrasts.

Example (only E switch):

Say the unblinded data is:

	$\mathcal{C}$	$\mathcal{A}$	ϕ	τ	ω
$\tilde{\mathcal{E}} = +1$	1	0.1	0.05	1	0.05
$\tilde{\mathcal{E}} = -1$	0.8	0.08	0.05	1	0.05

$$\mathcal{A}^{\mathrm{nr}} = 0.09$$

$$\mathcal{A}^{\mathcal{E}} = 0.01$$

$$\phi^{\mathrm{nr}} = 0.05$$

$$\phi^{\mathcal{E}} = 0$$

$$\mathcal{C}^{\mathrm{nr}} = 0.9$$

$$\mathcal{C}^{\mathcal{E}} = 0.1$$

$$\omega^{\mathrm{nr}} = 0.05$$

$$\omega^{\mathcal{E}} = 0$$

$$\mathcal{A}^{\mathrm{nr}}_{\mathrm{blinded}} = 0.09$$

$$\mathcal{A}^{\mathcal{E}}_{\mathrm{blinded}} = 0.01 + 0.01$$

$$\phi^{\mathrm{nr}}_{\mathrm{blinded}} = 0.05$$

$$\phi^{\mathcal{E}}_{\mathrm{blinded}} = 0 + \frac{0.01}{2\mathcal{C}^{\mathrm{nr}}}$$

$$\mathcal{C}^{\mathrm{nr}}_{\mathrm{blinded}} = 0.9$$

$$\mathcal{C}^{\mathcal{E}}_{\mathrm{blinded}} = 0.1$$

$$\phi^{\mathcal{E}=+1}_{\mathrm{blinded}} = 0.05 + \frac{0.01}{2\mathcal{C}^{\mathrm{nr}}} = 0.0\dot{5}$$

$$\phi^{\mathcal{E}=-1}_{\mathrm{blinded}} = 0.05 - \frac{0.01}{2\mathcal{C}^{\mathrm{nr}}} = 0.0\dot{4}$$

	$\mathcal{C}_{\mathrm{blinded}}$	$\mathcal{A}_{\mathrm{blinded}}$	ϕ_{blinded}(from $\mathcal{A}_{\mathrm{blinded}}$/ ϕ_{blinded})	τ_{blinded}	$\omega_{\mathrm{blinded}}$
$\tilde{\mathcal{E}} = +1$	1	0.11	$0.055/0.0\dot{5}$	1	$0.055/0.0\dot{5}$
$\tilde{\mathcal{E}} = -1$	0.8	0.07	$0.04375/0.0\dot{4}$	1	$0.04375/0.0\dot{4}$

This discrepancy reveals exactly how big the blind is

$$\mathcal{A}^{\mathrm{nr}}_{\mathrm{blinded}} = 0.09$$

$$\mathcal{A}^{\mathcal{E}}_{\mathrm{blinded}} = 0.01 + 0.01$$

$$\phi^{\mathrm{nr}}_{\mathrm{blinded}} = 0.05$$

$$\phi^{\mathcal{E}}_{\mathrm{blinded}} = 0 + \frac{0.01}{2\mathcal{C}^{\mathrm{nr}}}$$

$$\mathcal{C}^{\mathrm{nr}}_{\mathrm{blinded}} = 0.9$$

$$\mathcal{C}^{\mathcal{E}}_{\mathrm{blinded}} = 0.1$$

$$\phi^{\mathcal{E}=+1}_{\mathrm{blinded}} = 0.05 + \frac{0.01}{2\mathcal{C}^{\mathrm{nr}}} = 0.0\dot{5}$$

$$\phi^{\mathcal{E}=-1}_{\mathrm{blinded}} = 0.05 - \frac{0.01}{2\mathcal{C}^{\mathrm{nr}}} = 0.0\dot{4}$$

	$\mathcal{C}_{\mathrm{blinded}}$	$\mathcal{A}_{\mathrm{blinded}}$	ϕ_{blinded}(from $\mathcal{A}_{\mathrm{blinded}}$/ ϕ_{blinded})	τ_{blinded}	$\omega_{\mathrm{blinded}}$
$\tilde{\mathcal{E}} = +1$	1	0.11	$0.055/0.0\dot{5}$	1	$0.055/0.0\dot{5}$
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This discrepancy reveals exactly how big the blind is

“Inconsistent” blinding

- It was already known how to consistently blind in ACME II
- This consistent procedure was not chosen mostly to stay the same as ACME I. Also, it is arguably simpler conceptually.
- We propose changing the blinding procedure to this consistent procedure (applying blind in state basis looks equally simple conceptually)

“Inconsistent” blinding (For phase)

3 Example (issue with phase due to τ, exaggerated)

	$\phi^{\mathcal{NE}}$	τ	$\omega^{\mathcal{NE}}$
$\tilde{\mathcal{P}} = +1$	1	1	1
$\tilde{\mathcal{P}} = -1$	3	3	1

1. Blind using average $\tau = 2$ with $\omega^{\mathcal{NE}}_{\text{blind}} = 1$:

$$\phi^{\mathcal{NE}}_{\tilde{\mathcal{P}}=+1} = 1 \mapsto 1 + 1 * 2 = 3 \tag{4}$$

$$\phi^{\mathcal{NE}}_{\tilde{\mathcal{P}}=-1} = 3 \mapsto 3 + 1 * 2 = 5, \tag{5}$$

and if we use this ϕ to then compute ω,

$$\omega^{\mathcal{NE}}_{\tilde{\mathcal{P}}=+1} = \phi^{\mathcal{NE}}_{\tilde{\mathcal{P}}=+1} / \tau_{\tilde{\mathcal{P}}=+1} = 1 + 2 = 3 \tag{6}$$

$$\omega^{\mathcal{NE}}_{\tilde{\mathcal{P}}=-1} = \phi^{\mathcal{NE}}_{\tilde{\mathcal{P}}=-1} / \tau_{\tilde{\mathcal{P}}=+1} = 1 + 2/3 = 5/3. \tag{7}$$

The difference in effect of the applied blind basically reveal what the blind is since we know the value of τ.

2. Blind using super-block state dependent τ with $\omega_{\text{blind}}^{\mathcal{NE}} = 1$:

$$\phi_{\tilde{\mathcal{P}}=+1}^{\mathcal{NE}} = 1 \mapsto 1 + 1 * 1 = 2 \tag{8}$$

$$\phi_{\tilde{\mathcal{P}}=-1}^{\mathcal{NE}} = 3 \mapsto 3 + 1 * 3 = 6, \tag{9}$$

Hence, $\phi^{\mathcal{PNE}} = 1 \mapsto 2$.

In summary, if one uses method 1 to blind, this is mathematically similar to the issue of having asymmetry and using the non-reversing contrast. The main difference here is that asymmetry becomes phase and non-reversing contrast becomes average τ. The protocol is then: after blinding, one is not allowed to use phase to compute frequency. If one uses method 2 to blind, then one inevitably changes the values in the other parity channels, as it should be the case for state-by-state blinding.

Finding out the blind value when the blinded EDM is too big

Story from ACME II…

In ACME II, the blinded value of the EDM is quite big (sorry I forgot how big). This basically immediately told us that the blinded EDM consists mostly of the blind value, thus revealing a lot of information about the blind. This was a possible concern.

But really, we should not worry about how much information is revealed about the blind, but instead about how much is revealed about the unblinded EDM value.

Let us look at this more rigorously

We have random varaibles

1. E: value of the EDM, its distribution is somewhat arbitrary and describes one's belief about the real EDM value. Say it has pdf $f(e)$
2. B: value of the blind, its distribution is defined by us. Say it has pdf $g(b)$
3. $Y \equiv E + B$: sum of the EDM+blind visible by us. Its pdf is simply $\int f(y-b)g(b)db$ as E, B are independent

To see how much information is revealed by looking at the blinded EDM, we look at the conditional pdf of $E|Y$,

$$h_{E|Y}(e|y) = \frac{\mathrm{pdf}_{E,Y}(e,y)}{\mathrm{pdf}_Y(y)} = \frac{f(e)g(y-e)}{\int f(y-b)g(b)db}$$

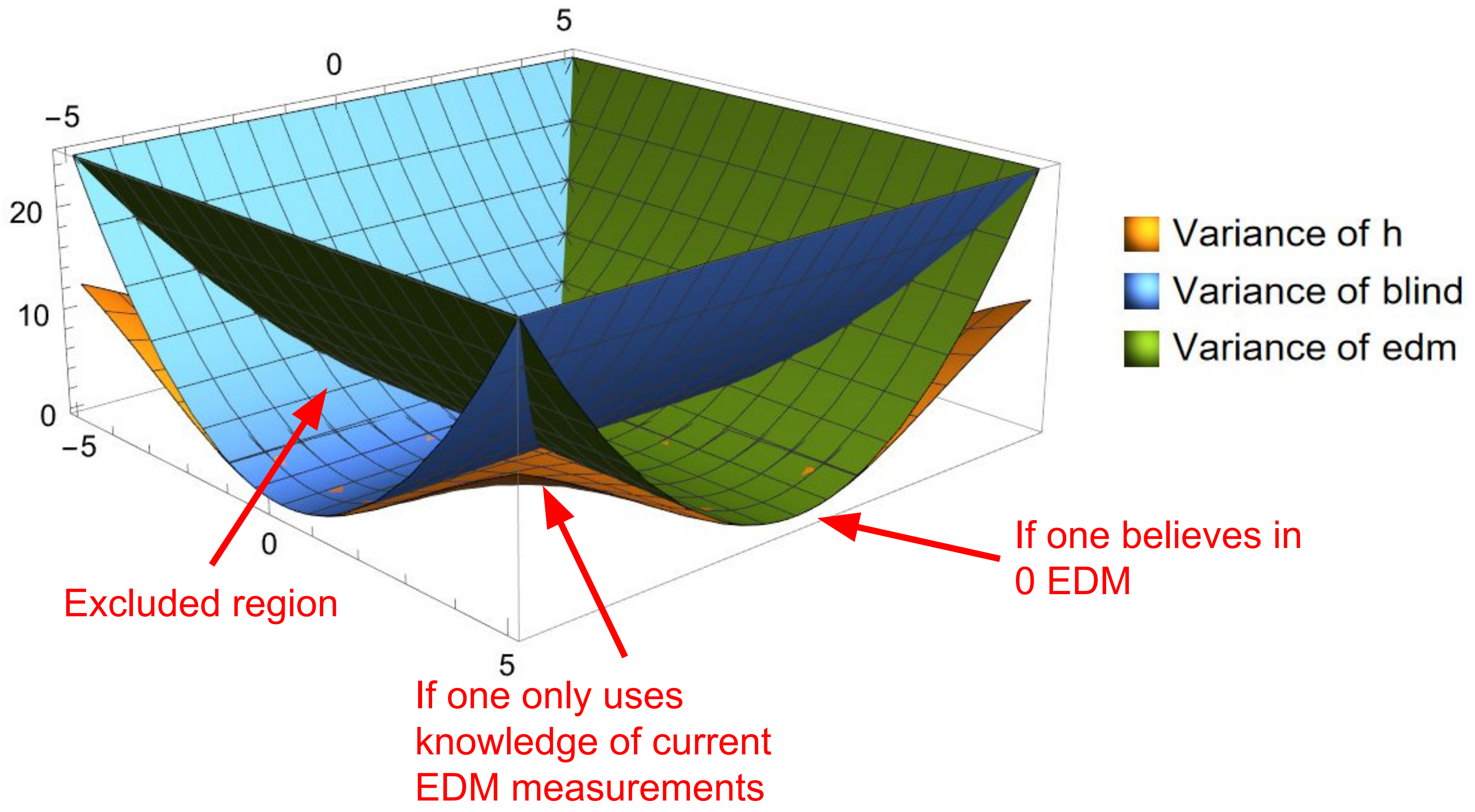
Let's look at an example where pdfs of E, B are Gaussians given by

$$f(e) = \frac{1}{\sqrt{2\pi\sigma_e}} e^{-\frac{e^2}{2\sigma_e^2}}$$
$$g(b) = \frac{1}{\sqrt{2\pi\sigma_b}} e^{-\frac{b^2}{2\sigma_b^2}}$$

It is straightforward to evaluate the conditional distribution

$$h_{E|Y}(e|y) = \frac{\sqrt{\frac{1}{\sigma_e^2} + \frac{1}{\sigma_b^2}}}{2\pi} e^{-\frac{(y\sigma_e^2 - e(\sigma_b^2+\sigma_e^2))^2}{2\sigma_b^2\sigma_e^2(\sigma_b^2+\sigma_e^2)}}$$

This pdf is a Gaussian with mean $\frac{y\sigma_e^2}{\sigma_b^2+\sigma_e^2}$ and standard deviation $\sqrt{\frac{\sigma_b^2\sigma_e^2}{\sigma_b^2+\sigma_e^2}}$


5
0
−5
20
10
0
−5
0
5
Variance of h
Variance of blind
Variance of edm
Excluded region
If one only uses knowledge of current EDM measurements
If one believes in 0 EDM