Molecular Asymmetry and Optical Cycling: Laser Cooling Asymmetric Top Molecules

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(Received 29 January 2020; revised 13 April 2020; accepted 1 June 2020; published 29 July 2020)

We present a practical roadmap to achieve optical cycling and laser cooling of asymmetric top molecules (ATMs). Our theoretical analysis describes how reduced molecular symmetry, as compared to diatomic and symmetric nonlinear molecules, plays a role in photon scattering. We present methods to circumvent limitations on rapid photon cycling in these systems. We calculate vibrational branching ratios for a diverse set of asymmetric top molecules and find that many species within a broad class of molecules can be effectively cooled with a manageable number of lasers. We also describe methods to achieve rotationally closed optical cycles in ATMs. Despite significant structural complexity, laser cooling can be made effective by using extensions of the current techniques for linear molecules. Potential scientific impacts of laser-cooled ATMs span frontiers in controlled chemistry, quantum simulation, and searches for physics beyond the Standard Model.

DOI: 10.1103/PhysRevX.10.031022 Subject Areas: Atomic and Molecular Physics

I. INTRODUCTION

Deviations from perfect symmetry play an important role in nature at a variety of spatial and temporal scales [1,2], ranging from the interactions of subatomic particles [3,4] to key biological processes like embryonic laterality [5,6]. Many important biochemical processes in the human body involve asymmetric, chiral molecules [7,8]. Despite their fundamental importance in other branches of science, asymmetric top molecules (ATMs, molecules with three distinct moments of inertia) have not been put under full quantum control. The traditional tools used to manipulate atomic and molecular samples with exquisite precision generally break down for ATMs due to intrinsic structural complexity.

Highly symmetric molecules have been brought under quantum control via recent advances producing laser-cooled molecules in the μK regime. These advances have spurred progress in such diverse fields as ultracold chemistry [9–11], quantum simulation of strongly correlated systems [12], and precision tests of fundamental physics [13,14]. Direct laser cooling of molecules is an important new tool for reaching the ultracold regime and achieving full control over the quantum state of the molecule. Furthermore, cooling allows for the long interaction times required to exploit these structures. (The technique of optoelectric Sisyphus cooling [15] is a complementary method to cool polyatomic molecules. However, it relies on neither directional momentum exchange between laser fields and molecules nor cycling optical photons, and is therefore orthogonal to the ideas described in this work.) SrF [16,17], CaF [18–21], and YO [22] have been directly laser cooled and trapped, and had their quantum states detected with near unit efficiency. The linear triatomic molecules SrOH [23,24], YbOH [25], and CaOH [26] have been laser cooled in one dimension, the first step to full quantum control.

Direct laser cooling requires a closed optical cycling transition such that more than $10^4$ photons may be scattered using a reasonable number of lasers and without significant loss to dark manifolds of states [27]. Previous laser cooling experiments relied on high molecular symmetry to greatly restrict losses from the optical cycle [28]. Extension to asymmetric molecules necessarily involves removing these restrictions. While Isaev and Berger previously performed ab initio estimates of the vibrational branching ratios for the chiral ATM MgCHDT [29], to date there has been no complete description of general methods to directly laser cool and trap ATMs. Previously, it had not been clear how the reduced symmetry of ATMs would affect optical cycling or even if full laser cooling of asymmetric species would be possible. Here, we present cases in which it is possible.

Although potentially challenging to produce, ultracold ATMs would offer many qualitatively unique features useful for a broad range of science. For example, ATMs can have up to three permanent dipole moments that could conceivably be controlled independently, a feature never

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present in higher symmetry species. Low-lying states with very long radiative lifetimes and large dipole moments promise strong molecule-molecule coupling in quantum simulation schemes [30–32]. In addition, asymmetric molecular structures often involve chemically and astrophysically interesting ligands [e.g., NH₂, SH, OC₂H₅, OCH(CH₃)₂, C₄H₅N, etc.] bonded to the optical-cycling-center atom. Finally, chiral molecules are, by necessity, ATMs. The determination of a general method to laser cool ATMs—allowing a ligand to be chosen primarily for its targeted utility rather than technological accessibility—would open up new avenues in a wide range of fields (see Sec. IV).

In this paper, we demonstrate that a general class of ATMs is amenable to laser cooling and trapping techniques. We focus on species of the form M-L, comprising an alkaline-earth atom (M) that is ionically and monovalently bonded to an electronegative ligand (L). In this case, the metal-centered unpaired valence electron acts somewhat independently of the bond, leading to good optical cycling properties [33,34]. We present calculations of the vibrational branching ratios in these molecules, demonstrating that with only a few repumping lasers one may scatter enough photons to slow, trap, and cool the molecules to the Doppler limit. Once at that temperature, these molecules will be generically amenable to sub-Doppler techniques, allowing access to the μK regime. Crucially, we describe how, despite the complex rotational structure in these asymmetric tops, a simple laser cooling scheme may be constructed that limits rotational branching to an easily repumped manifold of states. Using available spectroscopic data and new ab initio quantum chemistry calculations, we discover that, surprisingly, in many cases the required experimental complexity is not significantly greater than that encountered with linear polyatomic molecules.

II. ACHIEVING PHOTON CYCLING

Photon cycling, the repeated absorption and emission of photons, is a fundamental necessity to laser cooling (see Fig. 1). In this section, we discuss how the achievement of photon cycling is complicated by molecular structure. Here, we present techniques to overcome these complications.

Working within the Born-Oppenheimer approximation, we separate the discussion of electronic, vibrational, and rotational excitations. The possibility of Born-Oppenheimer approximation breakdown and its challenges are discussed in the Appendix A.

Laser cooling relies on photon cycling to generate dissipative forces [27,28]. Typically, about 10⁴–10⁵ photon scatters are necessary to apply sufficient force for slowing, cooling, and trapping. This case requires an excited state that decays preferentially to one ground state, with unplugged leaks to all other (possibly metastable) levels below about 1 part in 10⁴. The simplest atoms with this electronic structure are alkali or alkaline-earth (AE) atoms with one or two valence electrons in s orbitals. Electronic branching is then limited, or wholly eliminated, so that repumping from “dark” levels is easily achieved with at most a few additional laser frequencies. In the case of molecules, certain classes with Σ ground states have been found to be favorable to laser cooling due to their single valence electron in a metal-centered σ orbital [27,28]. Also identified are several Σ ground-state molecules with electronic structure that is promising for laser cooling [35,36]. However, it is not yet conclusively known whether polyatomic analogs of such species will have favorable optical cycling properties [37,38].

Vibrational and rotational substructure within each electronic state can be significant impediments to photon cycling in molecules. These internal degrees of freedom can change during an electronic transition, potentially leading to quick diffusion of population into “dark states” not coupled to the applied laser light. Molecules lost to these states will not be cooled further unless they are repumped, a task usually accomplished by application of additional laser or microwave frequencies. For linear molecules, angular-momentum selection rules can eliminate losses to dark rotational levels [39]. Losses to dark vibrational states are, by contrast, always problematic due to the general lack of selection rules. Vibrational branching during an electronic decay is governed probabilistically by Franck-Condon factors (FCFs), qᵣᵣ', which characterize the overlap between excited and ground vibrational wave functions. (Excited states are denoted by a single prime, while ground states are denoted by double primes.)
Vibrational losses are highly suppressed in molecules for which the ground- and excited-state potential energy surfaces resemble one another, as they do for AE metal atoms monovalently and ionically bonded to an electron-negative ligand (M\(^{+}\)L\(^{-}\)). In these species, electrostatic repulsion of the M\(^{+}\)-centered electron by the ligand anion induces orbital hybridization that moves the electron away from the chemical bond and strongly decouples electronic and vibrational excitations [40–42].

These ideas have led to the successful laser cooling of linear AE monofluorides and AE monohydroxides, including SrF [16,17,28,43–47], CaF [18–21,48,49], YbF [50], SrOH [23,24], YbOH [25], and CaOH [26]. Isaev and Berger proposed laser cooling molecules including CaCH\(_3\) and MgCH\(_3\) (and, notably, the ATM MgCHDT) in 2016 [29] but considered only the vibrational branching ratios. Recently, FCF calculations were reported for species with AE atoms bonded to hydrocarbon chains or fullerences, showing that these species have vibrational branching ratios that are favorable for laser cooling [51]. Reference [33] considered the issues of rotational and fine structure in symmetric, nonlinear molecules, finding that laser cooling symmetric top molecules requires at most one additional (rotational) repumper. For ATMs, it is not obvious a priori that typical laser cooling techniques will be applicable because, as the degree of symmetry of a molecule is decreased, selection rules become progressively weaker; many possible rotational- and vibrational-state changing decays are allowed during an electronic transition. Therefore, any discussion of photon cycling in ATMs requires careful consideration of both vibrational and rotational loss channels.

In ATMs, all three principal axes have distinct moments of inertia. These molecules can have at most twofold rotational symmetry, and formally, there can be no orbitally degenerate states. (Point groups C\(_1\), C\(_i\), C\(_s\), C\(_2\), D\(_2\), C\(_{2h}\), D\(_{2h}\), and C\(_{2v}\).) The reduced symmetry has several important consequences for laser cooling. First, mixing between electronic manifolds—especially among the closely spaced excited states \(\tilde{A}, \tilde{B}\), and \(\tilde{C}\)—may lead to perturbations that severely limit photon cycling. (It is conventional to label the ground electronic state as \(\tilde{X}\) and excited electronic states as \(\tilde{A}, \tilde{B}, ..., \) ordered by energy.) Second, vibronic selection rules that are present in the case of linear or symmetric top molecules may begin to break down. Third, the reduced symmetry requires careful consideration of how, and whether, a rotationally closed cycling transition can be constructed.

Here, we consider a very large class of molecules that has, fortunately, been the focus of considerable spectroscopic attention (making practical application in the laboratory much easier). Such species include (see Fig. 2) AE hydro-sulfides (MSH) [52–56], monoamides (MNH\(_2\)) [57–64], methanethiols (MSCH\(_3\)) [65], alkylamides (MNCH\(_3\)) [66], isopropoxides [MOCH(CH\(_3\))\(_2\)] [67], monoethoxides (MOC\(_2\)H\(_4\)) [68], pyrrollys (MC\(_4\)H\(_4\)N) [69], and methylpen-tadienyls (MC\(_5\)H\(_7\)) [69–71], where M = Mg, Ca, Sr, or Ba. (Yb-containing ATMs are interesting targets due to the high-Z nucleus, but have not yet been studied.) Chiral-substituted AE-methyls (MCHDT) [72–74] and AE-methoxides (MOCHDT) [75] are also considered. The wealth of spectroscopic data available for these species is essential to providing quantitative examples for the general approach to laser cooling ATMs.

### A. Electronic transitions

The structure of the low-lying electronic states of the ATMs considered here is best understood through their correlations with linear AE monohalides [40,60]. A schematic energy-level diagram is shown in Fig. 3. As the AE-metal atom M approaches the ligand L, one of the \(ns^2\) valence electrons is transferred from M to L. The electron remaining localized around M\(^{+}\) is polarized away from the bond via orbital mixing with excited \(np\) and \(nd\) orbitals [40]. In the linear limit, the ground state \(\tilde{X}\) is of \(\Sigma^+\) symmetry, while the two lowest electronically excited states are \(\tilde{A}^\Pi\) and \(\tilde{B}^\Sigma^+\). As the cylindrical symmetry of the molecule is broken, the degeneracy between orbitals directed along the \(b\) and \(c\) axes is lifted and the \(\tilde{A}^\Pi\) state splits in two.

In general, there will be four low-lying states of interest to laser cooling experiments, \(\tilde{X}^2\tilde{A}', \tilde{A}^2\tilde{A}', \tilde{B}^2\tilde{A}'\), and \(\tilde{C}^2\tilde{A}'.\) (The symmetry labeling will differ depending on the point group, but the overall structure will be similar.) For molecules of \(C_{2v}\) symmetry, the principal axes of the molecule coincide with the axes along which the electronic orbitals are aligned. For molecules of lower symmetry, there will be a misalignment between the M-L bond and the
primary axis along which the electronic orbitals are oriented. As will be discussed below, this angle determines to what extent the nominal rotational selection rules will hold. For the species considered here, the lowest three electronically excited states can be addressed by convenient laser wavelengths between 570 nm and 730 nm, and they have short lifetimes of about 20–40 ns. Both features are crucial for exerting large optical forces.

To understand the detailed electronic structure of laser-coolable ATMs, we have performed ab initio molecular structure calculations for four prototypical species: CaOH ($C_{\infty v}$ symmetry), CaCH$_3$ ($C_{3v}$ symmetry), CaNH$_2$ ($C_{2v}$ symmetry), and CaSH ($C_{1}$ symmetry). Figure 4 provides a comparison of the singly occupied molecular orbital (SOMO) and the lowest unoccupied molecular orbital (LUMO) as the symmetry of the ML molecule is reduced (column 1–4). Despite significant differences in structural symmetry, for the SOMO, LUMO, and LUMO + 1 (rows 1–3), the molecular orbitals are essentially unchanged as the molecular geometry is distorted. The orbitals are only slightly distorted for LUMO + 2 (row 4). Quantitatively, both ionic bonding and heavy valence electron localization on the metal atom are described by the Mulliken charge and spin population analyses. As the ligand is changed from OH ($C_{\infty v}$) to CH$_3$ ($C_{3v}$) to NH$_2$ ($C_{2v}$) and SH ($C_{1}$), the Mulliken charge on the Ca atom goes from +0.58 to +0.58 to +0.55 and +0.51 while the spin population changes from 0.999 to 0.908 to 0.982 and 0.963, respectively. This result is in qualitative agreement with the electron affinities determined for these anion ligands [77]. The similar electronic structure in these molecules suggests that laser cooling of ATMs will proceed similarly to cooling linear species.

Losses due to purely electronic transitions are not expected to affect photon cycling in these molecules. One potential loss channel involves radiative “cascade” decay through electronic states intermediate to the photon cycling states [22]. For the molecules considered here, no electronic states between the $\tilde{A}$ and $\tilde{X}$ states have been observed. When optical cycling with the excited $\tilde{B}$ or $\tilde{C}$ states, radiative cascades through the lower-lying states are suppressed by a factor of $(\Delta \omega / \omega)^{-3} \sim 10^4$ due to the small energy spacings. This rate is similar to those previously observed in experiments cooling diatomic species [18]. A second potential source of loss involves higher-order multipole transitions, e.g., M1 and E2 transitions, that would remove molecules from the optical cycle by populating opposite-parity rovibrational states. Such rates would be similar to those observed in previously cooled diatomic species, where losses due to M1 or E2 transitions have been
found to be negligible until more than $10^6$ photons have been scattered \[27,28,78\].

### B. Vibrational branching

Accurate values of the vibrational branching ratios during an electronic decay are of utmost importance to molecular laser cooling. To provide information relevant to laser cooling, these measurements would ideally use cw excitation to a low-$J$ rotational state, be free from state-changing collisions, include careful calibration of detection efficiency over a wide range of wavelengths, and have sensitivities at a level of less than $10^{-3}$. These restrictions severely limit the number of useful direct FCF measurements in the literature. By contrast (and fortunately), high-quality spectroscopic data, including bond lengths and angles for many molecules of interest, are plentiful. As such, we use a semiempirical approach to compute FCFs for a large set of ATMs that are expected to be favorable for laser cooling. When accurate high-resolution spectroscopic measurements are available, the computational methods described below can provide vibrational branching ratios in excellent agreement with observations and with accuracy better than purely \textit{ab initio} predictions \[79,80\]. The accuracy of these calculations depends entirely on the accuracy of the measured molecular geometries. In the absence of such experimental data, \textit{ab initio} calculations enable unique insights into identification of favorable ligands for laser cooling \[34\] and estimation of the vibrational branching ratios \[81\].

During a generic electronic decay, both the electronic and vibrational coordinates can vary. In many cases, one may neglect the dependence of the electronic transition moment on the nuclear coordinates \[82\]. For the molecules considered here, this approximation is justified by the excellent agreement between calculation and experiment (see Table I and Refs. \[79,80\]). In this approximation, the FCFs are computed by evaluating the overlap integral between excited and ground vibrational wave functions:

\[
q_{v',v''} = \left| \int \psi_{v'}(Q)^* \psi_{v''}(Q''') dQ'' dQ \right|^2, \tag{1}
\]

where $Q''$ represent the vibrational normal modes of the ground state and $Q'$ the vibrational normal modes of the excited state. We compute these integrals within the harmonic approximation, as anharmonic contributions are about $5 \times 10^{-2}$ for the molecules considered here \[65\].

We follow standard methods to calculate the FCFs \[84\]. In brief, we first perform a $GF$-matrix analysis \[85\] to determine the normal modes in each electronic manifold. Experimentally measured vibrational frequencies are used to fit the elements of the $F$ matrix. We include a Duschinsky transformation between ground and excited normal coordinates to account for normal mode mixing (see Fig. 5). We then follow the method of Sharp and Rosenstock \[86,87\] who used generating functions and a linear transformation between the normal coordinates of ground and excited states to define analytic expressions for the FCFs \[86,88\]. This model allows one to predict vibrational branching ratios for all vibrational modes within the harmonic approximation. It has been validated for many linear and nonlinear polyatomic species \[25,79,80\], with excellent agreement between theory and experiment. The computed FCFs depend strongly on the changes in molecular geometry upon electronic excitation and more weakly on changes in vibrational frequencies. High-resolution spectroscopy confirms that the molecular geometries change minimally upon electronic excitation \[57,59,60\].

We benchmark our multidimensional FCF calculations by comparing to experimental measurements. FCFs for SrNH$_2$ have previously been measured \[59\] on the $\tilde{C}^2A_1 \rightarrow \tilde{X}^2A_1$ and $\tilde{A}^2B_2 \rightarrow \tilde{X}^2A_1$ bands. Table I reports the calculated and measured vibrational branching ratios for SrNH$_2$. We find excellent agreement between theory and experiment. The small discrepancies, which occur in vibrations of the ligand, may arise from the fact that the spectroscopy is typically unable to fully determine the ligand geometry, so systematic errors affecting the calculations are possible. These calculations indicate that for SrNH$_2$, photon cycling of about $10^4$ photons should be possible with just three vibrational repumping lasers. There also exist measurements of vibrational branching ratios in CaOC$_2$H$_5$ \[68\], predicting a decay to the Ca-O stretching mode of about 0.1, in good agreement with our calculation below. (These measurements were taken with pulsed-laser excitation, and the data show signs of collisional excitation, which could present systematic errors in their interpretation.) Besides these few cases, vibrational branching ratios for nonlinear, laser-coolable species are rarely reported. Clearly, there is a need for new, accurate measurements of vibrational branching ratios in AE-containing ATMs.

Calculated multidimensional FCFs for CaNH$_2$, CaSH, and SrSH are reported in Table II. Although there are no published FCF measurements for these species, high-resolution studies have noted that off-diagonal decays...
were either unobservably small or highly suppressed, in agreement with our predictions [65]. For these molecules, our calculations indicate that 3–5 repumping lasers will be required to scatter more than $10^4$ photons. Vibrational normal modes of MSH species are shown in Fig. 5, depicting a strong mixing between the bending and M-S stretching motions; this mixing is included in our calculations. Dynamic visualizations of all vibrational normal modes for CaNH$_2$ are provided in the Supplemental Material [89]. A proper accounting of the differences between ground-state and excited-state normal modes is included in the calculations of Tables I and II. Comparison of the methods also shows that while the individual FCFs may differ slightly, the sum of the dominant few FCFs is consistent.

The computations above require the complete molecular geometry and vibrational frequencies for all vibrational modes, which have not been experimentally determined for many species. Often, only the metal-ligand bond lengths have been determined. Experience with the monohydroxides shows that the metal-ligand stretching mode is typically the dominant decay channel. Table III presents metal-ligand stretching-mode FCFs for a wide variety of species [90]. The FCFs for these ATMs are highly diagonal due to the small changes in potential energy surface shapes upon decay. We estimate the relative error in our calculations by using the same model to computing FCFs for isoelectronic diatomic and linear triatomic species; for the diagonal FCFs, the relative error is expected to be less than 3%. Note that perturbations in the excited electronic state can introduce unexpectedly strong off-diagonal decays [79], and for any particular species, the dominant uncertainty in the FCFs will likely be due to these perturbative couplings. They will become more prevalent, especially with increasing complexity and decreasing symmetry of the ligand (see Appendix A).

We have also conducted a set of \textit{ab initio} calculations to understand these species (see Supplemental Material [89]). Normal modes and vibrational frequencies were computed at optimized geometries using the ORCA Quantum Chemistry Software [92], and wave-function overlap integrals were computed numerically using the ezSpectrum software [93]. The calculations are benchmarked by testing the predictions for CaOH against experimentally determined FCFs [79]. For the lowest excited electronic transition $\tilde{A} \rightarrow \tilde{X}$, the FCF for the fundamental $0^0_0$ transition changed from 0.988 (CaOH) to 0.870 (CaSH) to 0.989 (CaNH$_2$). The dominant vibrational loss channel was to the Ca-ligand stretching mode for all three molecules with decay fractions 0.01 (CaOH), 0.10 (CaSH), and 0.01 (CaNH$_2$) to this mode. The Duschinsky transformation [94] of the normal modes was used for all three molecules to account for the change in normal coordinates in each electronic state. In the case of CaSH, we observed strong mixing between the stretching and bending motions (see Fig. 5). While the CaSH $0^0_0$ vibronic FCF is predicted to be less than 0.9, the sum of the three dominant vibrational loss channels is close to or greater than 0.99. Previous work [81] has indicated that only the sum of

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
Transition & Decay to & Calculated & Measured$^a$ \\
\hline
$\tilde{A} \rightarrow \tilde{X}$ & $0^0_0$ & 0.960 & 0.959 \\
& 1x Sr-N stretch & 0.039 & 0.04 \\
& 2x Sr-N stretch & $1 \times 10^{-3}$ & $1.6 \times 10^{-3}$ \\
& 1x NH$_2$ bend & $3 \times 10^{-5}$ & $2 \times 10^{-5}$ \\
& 1x N-H sym. stretch & $8 \times 10^{-6}$ & $4 \times 10^{-5}$ \\
\hline
$\tilde{C} \rightarrow \tilde{X}$ & $0^0_0$ & 0.976 & ... \\
& 1x Sr-N stretch & 0.011 & 0.01 \\
& 2x Sr-N stretch & $1 \times 10^{-4}$ & ... \\
& 1x NH$_2$ bend & 0.010 & overlapped \\
& 1x N-H sym. stretch & $6 \times 10^{-6}$ & $8 \times 10^{-6}$ \\
\hline
\end{tabular}
\caption{Comparison of measured and calculated vibrational branching ratios for SrNH$_2$. Because decay to the NH$_2$ bending mode is overlapped in the $\tilde{C} \rightarrow \tilde{X}$ band, we are not able to sum experimental FCFs in this case. The molecular geometry and vibrational frequencies used in the calculations are taken from Refs. [59,61,83].}
\end{table}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
Transition & Decay to & Calculated & Measured$^a$ \\
\hline
$\tilde{A} \rightarrow \tilde{X}$ & $0^0_0$ & 0.963 & 0.993 \\
& 1x Ca-N stretch & 0.034 & $2 \times 10^{-3}$ \\
& 1x NH$_2$ bend & $2 \times 10^{-3}$ & $1 \times 10^{-3}$ \\
& 2x Ca-N stretch & $1.6 \times 10^{-4}$ & $5 \times 10^{-5}$ \\
& 1x N-H sym. stretch & $3.1 \times 10^{-7}$ & $3 \times 10^{-6}$ \\
\hline
$\tilde{C} \rightarrow \tilde{X}$ & $0^0_0$ & 0.820 & 0.952 \\
& 1x Ca-S stretch & 0.163 & 0.0276 \\
& 1x Ca-S-H bend & 0.016 & 0.0199 \\
& 2x Ca-S stretch & $1.6 \times 10^{-4}$ & $4 \times 10^{-4}$ \\
& 1x S-H stretch & $1 \times 10^{-6}$ & $1 \times 10^{-6}$ \\
\hline
\end{tabular}
\caption{Calculated multidimensional FCFs for CaNH$_2$, CaSH, and SrSH. The molecular geometry and vibrational frequencies used in the calculations are as reported in Refs. [52–54,57,58,60,65].}
\end{table}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
Transition & Decay to & Calculated & Measured$^a$ \\
\hline
$\tilde{A} \rightarrow \tilde{X}$ & $0^0_0$ & 0.828 & 0.953 \\
& 1x Sr-S stretch & 0.170 & 0.046 \\
& 1x Sr-S-H bend & 0.0014 & $4 \times 10^{-4}$ \\
& 2x Sr-S stretch & $1 \times 10^{-4}$ & $< 10^{-5}$ \\
& 1x S-H stretch & $1 \times 10^{-6}$ & $1 \times 10^{-6}$ \\
\hline
\end{tabular}
\caption{Calculated multidimensional FCFs for SrSH.}
\end{table}
the three dominant FCFs should be considered stable for predictive purposes when purely \textit{ab initio} methods are being used.

Based on these calculations and their agreement with experiment, ATMs are strong candidates for optical cycling and laser cooling. For species of the form MSH and MNH$_2$ (M = Mg, Ca, Sr), the calculations indicate that at least one electronic transition in several of the species considered can scatter more than $10^3$ photons with only a single repumping laser. The calculations predict that the number of photon scatter per molecule is increased to more than $10^4$ per molecule—the typical number required to slow a molecular beam and capture into a MOT—with three repumping lasers. Indeed, the calculated FCFs are comparable to those of isoelectronic diatomic species that have already been successfully laser cooled [17,18]. For the larger species, at least 100 photons can be scattered with just two lasers. It is remarkable that even in species as complex as calcium methylcyclpentadienyl [CaC$_5$H$_4$CH$_3$, Fig. 2(g)], with 36 vibrational normal modes, the FCFs converge to close to or more than 0.95 with just one or two repumping lasers [70]. This level of convergence is sufficient for high-fidelity detection, preparation of single quantum states, or transverse laser cooling in order to enhance interrogation times in a molecular beam [24,50] (see Sec. III B).

### C. Rotational branching

Laser cooling requires, in addition to a vibrationally closed optical cycle, rotationally closed transitions. The rotational structure of ATMs can be extremely complex (see Appendix B). Following standard convention, we label the rotational states by $N_k K_c$. Here, $N$ is the total angular momentum excluding electron and nuclear spins. The principal rotational constants are labeled $A$, $B$, and $C$, where $A > B > C$. Note that $K_a$ and $K_c$ are not rigorous quantum numbers but are defined as the projection of $N$ onto the hypothetical symmetry axis formed by distorting the molecule to prolate ($a$ axis) and oblate limits ($c$ axis), respectively. The unpaired electron spin can couple weakly to rotation, and $J = N + S$ is the total angular momentum excluding nuclear spin. Hyperfine structure is typically unresolved ($< 1$ MHz). Losses to dark states throughout the complicated rotational ladder are a potential concern during photon cycling. Here, we show that rotational and parity selection rules can eliminate or mitigate these potential problems.

In a generic ATM, the transition dipole moment $\hat{\mu}$ can have components along any of the three principal axes. The transition dipole moment components are labeled $\hat{\mu}_a$, $\hat{\mu}_b$, and $\hat{\mu}_c$ to indicate their projection along the principal axes of the molecule. The transitions are then labeled $a$ type, $b$ type, or $c$ type according to which component of the transition dipole moment they couple. The selection rules for each type of transition are summarized in Table IV. There are additional

### TABLE III. Franck-Condon factors calculated for decay to 0 quanta ($q_{00}$) and 1 quantum ($q_{01}$) of the metal-ligand stretching mode. Here, $\lambda_{00}$ and $\lambda_{01}$ are the main cycling and first repumping transition wavelengths in nm. Experimentally measured geometries and frequencies come from Refs. [52–66,68–71,73,74].

<table>
<thead>
<tr>
<th>Molecule</th>
<th>Transition</th>
<th>$q_{00}$</th>
<th>$q_{01}$</th>
<th>$\lambda_{00}$ (nm)</th>
<th>$\lambda_{01}$ (nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CaSH</td>
<td>$A \rightarrow \tilde{X}$</td>
<td>0.826</td>
<td>0.157</td>
<td>650</td>
<td>664</td>
</tr>
<tr>
<td></td>
<td>$B \rightarrow \tilde{X}$</td>
<td>0.983</td>
<td>0.016</td>
<td>630</td>
<td>643</td>
</tr>
<tr>
<td>SrSH</td>
<td>$A \rightarrow \tilde{X}$</td>
<td>0.850</td>
<td>0.137</td>
<td>700</td>
<td>713</td>
</tr>
<tr>
<td></td>
<td>$B \rightarrow \tilde{X}$</td>
<td>0.962</td>
<td>0.037</td>
<td>675</td>
<td>687</td>
</tr>
<tr>
<td>MgNH$_3$$^a$</td>
<td>$A \rightarrow \tilde{X}$</td>
<td>0.933</td>
<td>0.065</td>
<td>$\sim 410^b$</td>
<td>$\sim 420^b$</td>
</tr>
<tr>
<td>CaNH$_2$</td>
<td>$A \rightarrow \tilde{X}$</td>
<td>0.964</td>
<td>0.035</td>
<td>640</td>
<td>662</td>
</tr>
<tr>
<td></td>
<td>$B \rightarrow \tilde{X}$</td>
<td>0.997</td>
<td>0.002</td>
<td>633</td>
<td>654</td>
</tr>
<tr>
<td>SrNH$_2$</td>
<td>$A \rightarrow \tilde{X}$</td>
<td>0.957</td>
<td>0.039</td>
<td>700</td>
<td>723</td>
</tr>
<tr>
<td></td>
<td>$B \rightarrow \tilde{X}$</td>
<td>0.971</td>
<td>0.028</td>
<td>679</td>
<td>700</td>
</tr>
<tr>
<td>BaNH$_2$$^c$</td>
<td>$A \rightarrow \tilde{X}$</td>
<td>0.916</td>
<td>0.081</td>
<td>$\sim 895^b$</td>
<td>$\sim 925^b$</td>
</tr>
<tr>
<td>MgCHDT</td>
<td>$A \rightarrow \tilde{X}$</td>
<td>0.966</td>
<td>0.062</td>
<td>499</td>
<td>526</td>
</tr>
<tr>
<td>CaCHDT</td>
<td>$A \rightarrow \tilde{X}$</td>
<td>0.997</td>
<td>0.002</td>
<td>680</td>
<td>700</td>
</tr>
<tr>
<td>CaOCHDT</td>
<td>$A \rightarrow \tilde{X}$</td>
<td>0.951</td>
<td>0.048</td>
<td>628</td>
<td>648</td>
</tr>
<tr>
<td>SrOCH$_2$</td>
<td>$A \rightarrow \tilde{X}$</td>
<td>0.945</td>
<td>0.046</td>
<td>565</td>
<td>586</td>
</tr>
<tr>
<td>CaSCH$_3$</td>
<td>$A \rightarrow \tilde{X}$</td>
<td>0.809</td>
<td>0.171</td>
<td>645</td>
<td>658</td>
</tr>
<tr>
<td>SrSCH$_3$</td>
<td>$A \rightarrow \tilde{X}$</td>
<td>0.832</td>
<td>0.153</td>
<td>694</td>
<td>706</td>
</tr>
<tr>
<td>CaNHCH$_3$</td>
<td>$A \rightarrow \tilde{X}$</td>
<td>0.953</td>
<td>0.046</td>
<td>652</td>
<td>673</td>
</tr>
<tr>
<td>SrNHCH$_3$</td>
<td>$A \rightarrow \tilde{X}$</td>
<td>0.944</td>
<td>0.054</td>
<td>706</td>
<td>726</td>
</tr>
<tr>
<td>CaOC$_2$H$_5$</td>
<td>$A \rightarrow \tilde{X}$</td>
<td>0.892</td>
<td>0.102</td>
<td>631</td>
<td>647</td>
</tr>
<tr>
<td>CaOCH(CH$_3$)$_2$</td>
<td>$A \rightarrow \tilde{X}$</td>
<td>0.956</td>
<td>0.043</td>
<td>632</td>
<td>645</td>
</tr>
<tr>
<td>SrOCH(CH$_3$)$_2$</td>
<td>$A \rightarrow \tilde{X}$</td>
<td>0.943</td>
<td>0.056</td>
<td>690</td>
<td>704</td>
</tr>
<tr>
<td>Ca$_3$H$_2$CH$_4$$^d$</td>
<td>$A \rightarrow \tilde{X}$</td>
<td>0.825</td>
<td>0.094</td>
<td>691</td>
<td>706</td>
</tr>
<tr>
<td></td>
<td>$B \rightarrow \tilde{X}$</td>
<td>0.86</td>
<td>0.078</td>
<td>686</td>
<td>701</td>
</tr>
</tbody>
</table>

* $^a$ Mg-N bond lengths inferred from Refs. [62,63,91].
* $^b$ Excitation wavelengths not measured at high resolution.
* $^c$ Ba-N bond lengths inferred from Refs. [34,64].
* $^d$ Dispersed fluorescence measured in Ref. [70].

### TABLE IV. Selection rules in the dipole approximation for the proposed ATM laser cooling transitions. Higher-order decays ($\Delta K_c = \pm 2$, etc.) are suppressed but may be induced by perturbations. There is an additional restriction due to the fact that $K_a + K_c = N$ or $N + 1$ [95,96].

<table>
<thead>
<tr>
<th>Type</th>
<th>$\Delta K_a$</th>
<th>$\Delta K_c$</th>
<th>Exceptions</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$ type</td>
<td>$0$</td>
<td>$\pm 1$</td>
<td>$\Delta N \neq 0$ for $K'_a \rightarrow K''_a = 0$</td>
</tr>
<tr>
<td>$b$ type</td>
<td>$\pm 1$</td>
<td>$\pm 1$</td>
<td>$\Delta N \neq 0$ for $K'_b \rightarrow K''_b = 0$</td>
</tr>
<tr>
<td>$c$ type</td>
<td>$\pm 1$</td>
<td>$0$</td>
<td>$\Delta N \neq 0$ for $K'_c \rightarrow K''_c = 0$</td>
</tr>
</tbody>
</table>
bands shown here are similar to a applicable to the branch designation closure on a the first letter indicates the appropriate Hund shown in Fig. 6(a). The rotational structure is reminiscent of a restrictions due to the fact that $K_a + K_c = N$ or $N + 1$ [95,96]. The selection rules and their symmetry considerations are discussed in detail in Appendix C.

Using the selection rules enumerated in Table IV, we find that it is possible to construct closed cycling schemes for transitions coupling to any component of $\tilde{\mu}$. Some representative photon cycling schemes are presented in Fig. 6. Dashed lines indicate rotational branches, which may be induced by spin-rotation and asymmetry doubling are approximately the same size. These level diagrams assume a near-prolate ATM, but analogous results apply for near-oblate tops.

FIG. 6. Schemes to achieve rotationally closed photon cycling for pure (a) $a$-type transitions, (b) $b$-type transitions, and (c) $c$-type transitions. Dashed lines indicate transitions that will need to be repumped in special cases, as described in the text. In the $K_x = 1$ levels, the spin-rotation and asymmetry doubling are approximately the same size. These level diagrams assume a near-prolate ATM, but analogous results apply for near-oblate tops.

transitions in linear species. (Such a transition is proposed for laser cooling the CH radical [97].) Here, we choose to drive the $P_{Q_{12}}(1, 0, 0.5)$ and $P_{P_{11}}(1, 0, 1.5)$ lines. Similar to the $a$-type band, a rf sideband is required to address the two spin-rotation components of the $N'' = 1$ manifold for either $b$-type or $c$-type bands. Note that the $K''_a = 1$ state, which is convenient for optical cycling, provides a suitable ground state because it is metastable with lifetimes greater than 10 s for the species considered here.

The cycling schemes in Fig. 6 provide a basis for rotational closure in realistic molecules. Roughly speaking, $\tilde{A} \rightarrow \tilde{X}$ decays follow $b$-type selection rules, $\tilde{B} \rightarrow \tilde{X}$ decays obey $c$-type selection rules, and $\tilde{C} \rightarrow \tilde{X}$ decays obey $a$-type selection rules. As the molecule becomes more asymmetric, these patterns will break down, and a given excited state may decay via one of several components of $\tilde{\mu}$. A hybrid $ab$-type band will commonly occur for molecules with $C_s$ symmetry due to the misalignment of the principal axes from the electronic orbitals [see Figs. (4(m)–4(p)]. One additional decay must be repumped, in essence combining the schemes in Figs. 6(a) and 6(b). Alternatively, microwave remixing within the ground manifold can return population to the optical cycle.

The degree of asymmetry will determine the relative strengths of different components of $\tilde{\mu}$. For example, because $\mu_a \gg \mu_c$, when photon cycling on the $\tilde{C} \rightarrow \tilde{X}$ band of hydrosulfide species, these can scatter about $10^4$ photons using the scheme of Fig. 6(a) before repumping of the $K''_a = 1$ state is required (see Appendix C and Supplemental Material [89]). For hybrid $bc$-type decays, which may occur in molecules with symmetry lower than $C_s$ or due to perturbations, repumping is easily achieved by combining the schemes of Figs. 6(b) and 6(c). A single laser with sidebands imposed by EOMs or AOMs can bridge the full spin-rotation or asymmetry doubling structure. Finally, for species with near-$C_{3v}$ symmetry (e.g., MOCHDT or MCHDT), where the $\tilde{A}^2A'$ and $\tilde{B}^2A''$ states have essentially coalesced into the $\tilde{A}^2E$ level, an additional decay to the $2_{1K} (J'' = 3/2)$ state will appear. Again, this state can be remixed using either microwaves or a single additional laser frequency, as shown by dashed lines in Fig. 6. The effects of perturbations, though not expected to significantly alter the results presented above, are discussed in Appendix C.

Overall we see that, despite the complex ATM rotational structure, application of one or two lasers (depending on the molecular species) with appropriate rf sidebands leads to rotational closure sufficient for scattering close to or more than $10^4$ photons. Remarkably, this level of complexity is comparable to that required for laser cooling of diatomic species for which rotational repumping is implemented [17,22,47,98].

D. General principles

The detailed calculations of the previous section focused on select species that have been observed in prior
experiments. These calculations elucidate many general principles that apply to the whole class of ATMs with optical cycling centers.

First, the key features of the electronic structure derive from the metal-centered valence electron for which potential energy surfaces vary little between electronic states. Our calculations show that the electronic distribution is only weakly dependent on the structural asymmetry of the ligand. Molecules containing other AE or AE-like atoms, or other ligands, should have similar electronic structure and be favorable for laser cooling. Certain AE (or AE-like) atoms may be ideal for particular applications (see Sec. IV). For instance, the low mass of Be could be beneficial to studies involving torsional modes, while the high-Z Yb nucleus can enhance symmetry-violating effects. Beyond molecules with AE atoms, several laser-coolable molecules have been identified that contain cycling centers from group 3 or 13 of the periodic table, notably TiO [39], AlF [35], TIF [36], and BH [99]. Isoelectronic ATM species (AISH, TIOCH3CH3, etc.) may also have highly diagonal FCFs. However, to date, such species have received little or no spectroscopic attention.

Second, although the FCFs do not follow selection rules present for linear species, they are still highly diagonal. In general, the off-diagonal decay strengths do not increase as the molecule becomes “more” asymmetric. Rather, the strengths remain weak so long as the potential energy surfaces are similar in the ground and excited states. Based on the geometry changes measured by high-resolution spectroscopy and predicted by our ab initio calculations, this is the case for the broad class of molecules explored here. While a dedicated vibrational analysis will be required for any particular molecule, our calculations indicate that dozens of ATM species are highly amenable to photon cycling. Vibrational anharmonicity is small for the molecules considered here but may be important for “floppier” ATMs of interest in the future. Torsional modes present in many ATMs, e.g., those with off-axis methyl groups, may require repumping when cycling more than 10^3 photons. Measurements indicate that decays to such vibrations are strongly suppressed (in the case of CaC2H5CH3) or unobservably small (in the case of CaOC2H5) [68,70], as expected by the intuition provided above. Note that torsional modes can be repumped just as any other vibrational mode, so they do not pose a fundamental impediment to laser cooling. Finally, as the molecule size increases, one may begin to worry about internal vibrational redistribution (IVR) due to a high density of states. Even the largest molecules proposed in this work, e.g., CaC2H5CH3 with 36 vibrational modes, do not show evidence of IVR in dispersed fluorescence measurements [70]. Nonetheless, for sufficiently large species, it will be important to reconsider whether IVR is a potential loss channel.

Third, we have seen how the leakages due to rotational branching can be mitigated or wholly eliminated by working with low-J states and approximate rotational selection rules. While we primarily considered the near-prolate limit, molecules near the oblate limit behave analogously but with K_a and K_c interchanged. Even for chiral molecules near the prolate (or oblate) limit, the same rotational closure schemes are expected to be applicable. The strength of hybrid rotational branches is expected to increase with the degree of structural asymmetry. Additionally, off-diagonal terms in the rotational Hamiltonian (see Appendix B) may admit levels with \( \Delta K_a = \pm 2 \) and \( \Delta K_c = \pm 2 \) selection rules. However, these matrix elements conserve \( J \), so for the low-J states considered here, the Hamiltonian is still highly restricted. Transitions with \( \Delta K_a = \pm 2, \pm 3, \ldots \) (and similarly for \( \Delta K_c \)) are limited by the restriction that \( K_a + K_c = N \) or \( N + 1 \). In the absence of strong perturbations, even highly asymmetric species will likely require at most one or two additional laser or microwave frequencies for rotational closure. The rotational-level structure for any given molecule will be determined by the particular values of A, B, and C, and the levels will not necessarily group into nearly spaced sets of levels labeled by \( K_a \) (prolate) or \( K_c \) (oblate).

### III. EXPERIMENTAL OUTLOOK

#### A. Molecule production

The molecules considered above can be generated using the cryogenic buffer-gas cooling method, which has been used in all molecular laser cooling experiments to date [27,28,100]. This method involves ablation of a metallic target into a reactant gas, followed by buffer-gas cooling due to collisions with cryogenic He, as demonstrated with Ca + SF_6 [18] and Yb + CH3OH [25]. Commercially available reactants such as hydrogen sulfide, ammonia, chiral methyl fluoride, thiols, pentadienyls, etc. can be introduced with a heated capillary to produce the species considered above. Typical yields of about 10^10 molecules per single rotational state per ablation pulse have been demonstrated for a variety of molecules [100]. Production can be enhanced by several orders of magnitude by exciting the ablated metal atoms to the metastable 3P_1 state prior to reaction [65,66,101].

Especially for larger molecules, buffer-gas cooling provides invaluable phase-space compression and limits the number of populated rovibrational modes such that significant molecular population is available for manipulation. For the molecules considered here, rotational constants fall in the range \( A \sim 1–10 \text{ cm}^{-1} \), \( B \sim 0.05–0.5 \text{ cm}^{-1} \), and \( C \sim 0.05–0.5 \text{ cm}^{-1} \). At typical buffer-gas temperatures of 2 K, nearly 50% of the produced molecules will be cooled into the rotationally closed levels. In addition, vibrational thermalization with the buffer gas can be reasonably efficient, especially for low-frequency vibrational modes [102,103]. Based on these previous studies of buffer-gas cooling, we expect more than 90% of the
produced molecules to be cooled into the few vibrational states that require laser repumping and will therefore be included in the optical cycle naturally. Together, this leads to about $10^9 - 10^{10}$ molecules per shot.

Rotational cooling is an important consideration when deciding on the exact photon cycling scheme to employ. For species without reflection symmetry, population will be cooled into the lowest $K''_u = 0$ state, with a smaller amount of population in $K''_u = 1$. On the other hand, molecules with $C_2$ symmetry will cool into both the $K''_u = 0$ and $K''_u = 1$ levels due to nuclear spin statistics, as previously observed in supersonic beams [61]. Because populating the $K''_u = 1$ level is often useful for photon cycling, we emphasize that it is straightforward to populate this level for any of the species discussed here using microwaves, optical pumping, or natural population.

B. Photon cycling to laser cooling

To maximize the magnitude of the optical forces exerted during optical cycling, it is desirable to maximize the scattering rate $R_{sc}$. Molecules often exhibit low scattering rates ($\sim 3 \times 10^7$ s$^{-1}$) because vibrational repumping requires coupling a large number of ground states to the same excited level [17]. The presence of numerous excited vibronic states provides a means to “break up” these bottlenecks, as done previously for linear molecules [24–26]. Taking into consideration the necessary rovibrational repumping and the excited states available for repumping, the achievable photon scattering rate for ATMs falls in the range $R_{sc}^{\text{max}} \approx (0.1–0.25)\Gamma$ [17]. This rate is comparable to the scattering rates achieved in recent experimental demonstrations of molecular laser cooling [17,26].

Intriguingly, ATMs may have somewhat large capture velocities in molecular MOTs as compared to linear species. Reference [104] showed that the magneto-optical force, and therefore capture velocity, of a typical magneto-optical trap (MOT) grows with the increase of the excited state $g$ factor. Small excited $g$ factors in the $A^2\Pi$ state ($\sim 10^{-2}$) have commonly limited capture velocities to about 5–10 m/s [48,78]. Zeeman tuning in the excited states of ATMs discussed here can, in general, be much larger ($g_L \Lambda + g_S \Sigma \sim 0.1–1$) due to quenching of the orbital angular momentum [105]. This tuning would increase the capture velocity of a molecular MOT for ATMs to more than 10 m/s, greatly increasing capture efficiency.

IV. APPLICATIONS

Applications of laser-cooled ATMs span a wide range of physical and chemical research. We highlight here a few promising research directions where molecular asymmetry adds valuable qualitative and quantitative advantages.

A. Quantum computation and simulation

Ultracold molecules with large electric dipole moments are an exciting new platform for quantum simulation and quantum information processing [106–111] and quantum information processing [31,112–115]. ATMs such as CaSH or SrSH are strongly suited for these experiments due to their small parity doublets, very large permanent dipole moments, and complex internal structure.

The complex internal structure of ATMs allows robust encoding of quantum information, as recently proposed in Ref. [116]. The structural asymmetry, present in ATMs but not in higher-symmetry species, is a crucial part of the proposed qubit encoding scheme. Additionally, the rich internal structure of ATMs will allow robust use of qudits, higher-dimensional analogs of two-level qudits [32]. This method could enable improved error correction [117–120], larger Hilbert spaces with fewer physical particles, and faster computation times. The rotational levels in ATMs serve as qudits, similarly to recent proposals for diatomic species [32] but with additional advantages. Importantly, not only are there more low-lying (metastable) rotational states to manipulate, but many more of these levels are coupled to one another by dipole-allowed transitions [121].

Additionally, ultracold ATMs are a promising platform for quantum simulation or information processing due to their strong dipole-dipole interactions. For example, CaSH has a large molecule-frame dipole moment of about 5.5D along the $a$ axis and an additional dipole moment of about 1.5D along the $b$ axis [30] (the bent structure is an important contributor to this larger $a$-axis dipole moment, as can be seen by comparing the cases of CaSH ($d \approx 5.5$ D) and CaOH ($d \approx 1.4$ D) [30,122]). The diatomic species cooled so far only have significantly smaller permanent dipole moments, but they also cannot be fully polarized with reasonable laboratory fields. ATMs therefore offer a viable route to achieving molecule-molecule couplings 1–2 orders of magnitude stronger than can be achieved with laser-cooled diatomic species to date [28]. Figure 7 shows the lab-frame dipole moments of CaSH in several rotational states; note that the molecule is fully polarized at applied fields close to or less than 100 V/cm. The presence of states with effectively zero dipole moment projection is also useful for “shelving” [115]. Such states arise from the parity-doublet structure, which is generically present in ATM molecules due to their structural asymmetry.

B. Searches for physics beyond the Standard Model

Searches for the electron electric dipole moment (eEDM) have provided increasingly tight constraints on physical theories beyond the Standard Model at energies of about 10 TeV [123–127]. A new generation of experiments searching for nuclear magnetic quadrupole moments (nMQMs) and nuclear Schiff moments are expected to probe similar energies for new particles leading to $T$, $P$-violating nuclear forces [128]. The precision of such experiments is typically optimized by maximizing the effective internal electric field, the number of molecules probed, and the coherence time of the experiment [129].
ATMs containing high-mass nuclei (e.g., MSH or MNH$_2$ with M = Ra or Yb) combine all of these advantages. Compared to the degenerate vibrational bending modes in linear molecules, which have been of recent interest [14,81,130], the parity doublets in ATMs arise from very low-lying rotational structure and therefore have radiative lifetimes longer by a factor of about 100–1000. This case will lead to eEDM or nMQM sensitivity enhanced by 1–2 orders of magnitude. For example, an eEDM under the same conditions as proposed in Ref.[14] but using YbSH instead of YbOH would yield an eEDM (statistical) sensitivity of about $10^{-33}–10^{-34}$ e cm, which is about 4 orders of magnitude beyond the current limit [124]. Other theories beyond the Standard Model predict variations of “fundamental” constants such as the proton-to-electron mass ratio, $\mu = m_p/m_e \sim 1836$ [131].

Large-amplitude motions in polyatomic species, such as inversion and hindered rotation, yield orders-of-magnitude enhanced sensitivity to $\mu$ variation [132–135]. These motions are present in any molecule containing an internal rotor (e.g., CH$_3$ group) but have no analog in linear or symmetric top molecules. Table III includes the laser-coolable analogs of these molecules, e.g., CaSCH$_3$. If lower mass is desired, Be or Mg atoms could be substituted in place of Ca. Similarly, CaOC$_2$H$_5$, CaC$_2$H$_4$CH$_3$, and CaNHCH$_3$ offer methyl groups with hindered internal rotations that may offer large enhancement factors for tests of $\mu$ variation. Efficient state preparation, unit-efficiency readout, phase-space compression for enhanced counting statistics, and long interrogation times enabled by laser cooling may make these systems ideal test beds for probing variation of fundamental constants in the laboratory.

C. Ultracold chiral species

Photon cycling and laser cooling of chiral molecules provide fascinating opportunities for chiral separation and measurement of the slight energy difference between enantiomers, potentially elucidating some aspects of the single chirality of many biological molecules [7,8]. The parity-violating energy difference between enantiomers leads to slight differences in the rovibrational spectra of left- and right-handed species, which can be probed with molecules such as, e.g., CaOCHDCH$_3$ or YbOCHDT. The long interrogation times enabled by traps will greatly enhance the precision of searches for fundamental parity violation, where precision less than 1 Hz on vibrational intervals is typically required [136]. At the same time, the complex rovibrational spectrum of ATMs, including $K$ and $\ell$ doublets as well as common near degeneracies between modes with very different vibrational and rotational composition, will provide a rich spectrum with which to perform the requisite frequency measurements. Furthermore, one could attempt to combine photon cycling and demonstrated chiral detection and state transfer techniques [137] to create purified samples of a single enantiomer, either in a beam or inside an optical trap. Not only would this be a powerful new method of chiral separation, but it would also potentially help suppress systematic errors in such precision measurements.

D. Cold chemistry and collisions

The realization that ATMs can be laser cooled opens up the possibility of bringing a more diverse array of chemical constituents to the ultracold regime. A combination of recent, unprecedented control of molecular dissociation [138] with theoretical studies proposing to produce ultracold atoms in this way [139,140] could be used to produce chemically relevant molecular fragments such as SH, SCH$_3$, S, NH$_2$, C$_4$H$_4$CH$_3$, or OC$_2$H$_5$ with even lower kinetic energy distributions to their (laser-coolable) parent molecules. The production methods described in Sec. III A can be readily adapted to produce more exotic chalcogen-containing...
polyatomic molecules, e.g., Se- or Te-containing molecules. In addition, this work demonstrates the possibility to cool “half-sandwich” molecules containing unsaturated carbon rings (e.g., CaC₅H₂CH₃). These species represent a versatile playground for organometallic chemistry due to the ability to synthesize species with unique substituents around the ring. Laser cooling of AE-bonded hydrocarbon rings, followed by bond-selective dissociation, represents a plausible way to generate samples of isolated, cold carbon rings. This method is particularly well suited to hydrocarbons, which possess neither electric nor magnetic dipole moments (or very small ones) and are therefore difficult to manipulate in the gas phase without the addition of an optical cycling center. The realization that this breadth of chemical bonds is amenable to photon cycling will also likely aid attempts to functionalize surfaces by deposition of optical cycling centers on a substrate while preserving the optical cycling properties [141].

Another interesting avenue is the study of fundamental collisional processes. Because of their geometrical asymmetry, ATMs can possess up to three distinct electric dipole moments and could be fully oriented using a combination of dc and microwave fields. This avenue will open up new possibilities of diatomic molecules where at most one electric dipole moment and could be fully oriented using a combination of dc and microwave fields. This avenue will open up new avenues to develop a rational basis for molecule selection [34]. Recent work has shown that vibronic coupling mechanisms can be evaluated for linear molecules with quantitative accuracy [152]. Extensions of both results to nonlinear, asymmetric species are now desirable.

The experimental outlook for laser-cooled asymmetric top molecules is quite bright, thanks in part to the ability to produce large numbers of these molecules in cryogenic buffer-gas beams. Our hope is that this work will spur robust theoretical and experimental exploration, similar to early proposals on diatomic molecules with optical cycling centers.

V. CONCLUSION

In this paper, we analyze how the discrete breakdown of structural symmetry in polyatomic molecules influences repeated optical cycling and direct laser cooling, two cornerstones of modern atomic and molecular physics. We show that for a generic class of asymmetric top molecules, optical cycling and laser cooling are possible. Our calculations, using both semiempirical and ab initio methods, demonstrate that a wide array of metal-ligand bonds have favorable optical cycling properties, introducing nitrogen- and sulfur-containing asymmetric compounds as laser cooling candidates. We also discuss how some of the approximations used in these calculations may break down, while showing that this should not significantly hamper the optical cycling possibilities.

New experimental measurements of the vibrational branching ratios would further benchmark our calculations. Measurements accurate at the 10⁻³ level or better will be necessary to assess the viability of particular laser cooling candidates, an accuracy possible with current spectroscopic techniques [151]. Improved theoretical calculations will also aid searches for species suited to laser cooling. Ab initio methods have previously been applied to linear molecules to develop a rational basis for molecule selection [51]. Where they used ab initio methods to calculate Franck-Condon factors for various polyatomic molecules with increasing complexity.

Very recently, direct laser cooling has been extended to the symmetric, nonlinear molecule CaOCH₃ [153].

APPENDIX A: BORN-OPPENHEIMER APPROXIMATION BREAKDOWN

We have generally been working within the Born-Oppenheimer (BO) approximation, in which potential energy surfaces are assumed to be well separated and noninteracting. In reality, vibronic coupling between these adiabatic surfaces can affect this picture [154]. BO approximation breakdown is especially likely as the size of the molecule grows because the number of states and the probability of near degeneracies increase. Furthermore, breakdown of the BO approximation is far more likely in the excited electronic states ̅A, ̅B, and ̅C due to their close proximity.
BO approximation breakdown can alter the intensities of rovibronic decays as compared to those calculated above. This result is not necessarily detrimental to photon cycling and laser cooling, so long as the branching induced by vibronic coupling is contained within a small number of modes that are easily repumped. Whether this is the case will require a careful, specialized analysis for any given molecule of interest. Here, we outline general features of such analyses.

The effects of vibronic coupling can be described by introducing a term $H_{ev}$ to the Hamiltonian, which mixes two (unperturbed) electronic states due to the influence of vibrations. It is natural to expand this perturbation term in the normal coordinates, yielding, to first order [155],

$$H_{ev}(x, Q) = H_{ev}^0(x, 0) + \sum_{j} \left( \frac{\partial H_{ev}}{\partial Q_j} \right)_0 Q_j,$$

(A1)

where $x$ are the electronic coordinates and $Q_j$ the vibrational normal modes. The physical content of Eq. (A1) is that vibronic perturbations can be introduced due to direct coupling between two vibronic states (the zeroth-order term) or by a vibrational “promoter” mode (the first-order term).

Because the Hamiltonian belongs to the totally symmetric representation, there are restrictions on which vibronic levels can interact under the influence of Eq. (A1). For instance, because $H_{ev}^0(x, 0)$ is totally symmetric, it can connect the $A^2A'$ and $C^2A'$ states in $C_j$ symmetry. It cannot connect any of the three lowest electronically excited states ($A^2B_2$, $B^2B_1$, and $C^2A_1$) in $C_{2v}$ symmetry because all three of these states have different symmetry.

The first-order term in Eq. (A1) is a linear sum over the normal modes, so we consider each term in the sum individually. The product $(\partial H_{ev}/\partial Q_j)_0 Q_j$ must be totally symmetric, so the symmetry of $(\partial H_{ev}/\partial Q_j)_0$ can be determined once the normal-mode symmetries are known. As an example, in $C_{2v}$, a vibrational mode with symmetry $a_2$ can induce coupling between vibronic states with symmetries $A_2$ and $B_1$.

While these considerations can help predict the propensity for BO approximation breakdown, they do not provide any estimate of the breakdown’s magnitude. A simple estimate for the impact of BO approximation breakdown can come from the case of linear molecules that have previously been laser cooled. Typical perturbation matrix elements are in the range of about 0.5–5 cm$^{-1}$ for CaOH or SrOH when Renner-Teller vibronic interactions are considered [156,157]. Taking this scale as the typical matrix element, the strength of vibronic interactions can be estimated using second-order perturbation theory and typical energy splittings in the excited electronic states. Such splittings are typically about 100–1000 cm$^{-1}$, implying that BO breakdown can induce unexpected decays at the $10^{-3}$–$10^{-6}$ level. In the case of very near degeneracies or enhanced coupling matrix elements, this interaction can be much stronger.

**APPENDIX B: ROTATIONAL STRUCTURE**

The rotational Hamiltonian for an ATM in the principal axis system, including the spin-orbit interaction, can be expressed as [60,158]

$$H = A(N_a - L_a)^2 + B(N_b - L_b)^2 + C(N_c - L_c)^2 + \sum_j \xi \ell_j \cdot s_j,$$

(B1)

where $\hat{N}$ is the total angular momentum excluding spin, $\hat{L}$ is the electronic orbital angular momentum, $\xi$ characterizes the spin-orbit interaction, and the sum over $j$ includes all electrons in the molecule. Following Refs. [54,60], Eq. (B1) can be reduced to a purely rotational Hamiltonian,

$$H_{rot} = AN_a^2 + BN_b^2 + CN_c^2,$$

(B2)

with perturbations arising from cross terms,

$$H' = -2(AN_aL_a + BN_bL_b + CN_cL_c) + \sum_j \xi \ell_j \cdot s_j.$$

(B3)

A Van Vleck transformation produces an effective spin-rotation Hamiltonian [54,60,158]

$$H_{sr} = \frac{1}{2} \sum_{\alpha \beta} \epsilon_{\alpha \beta} (N_{\alpha} S_{\beta} + S_{\beta} N_{\alpha}),$$

(B4)

which operates only within a given electronic state. The elements of the spin-rotation tensor, $\epsilon_{\alpha \beta}$, contain first-order contributions due to direct coupling between electron spin and molecular rotation and second-order contributions due to cross terms with the spin-orbit interaction. These second-order contributions are often a large portion of the $\epsilon_{\alpha \beta}$ components. While $\epsilon_{\alpha \beta}$ has nine possible parameters, for molecules with orthorhombic symmetry, only the three parameters $\epsilon_{aa}$, $\epsilon_{bb}$, and $\epsilon_{cc}$ are nonzero. The rotational constants are also contaminated by second-order contributions, although these are generally smaller in magnitude than the first-order (rigid-body rotation) terms. We emphasize that careful perturbation of the constants is important to assign the accurate geometries necessary for FCF calculations.

For the molecules considered in this paper, the hyperfine structure is negligible when the only nuclei with nonzero spin are located multiple bond lengths away from the valence electron. (To date, only spin-0 Ca and Sr nuclei have been used in molecular laser cooling experiments.) If fermionic metals are considered as the optical cycling center, these hyperfine terms must be added, and the analysis proceeds as a straightforward extension from that presented in this work. Hyperfine interactions can mix
rotational states with selection rules $\Delta N = \pm 2$, $\Delta F = 0$ [159,160]. However, these couplings are very weak [160] and have not affected the laser cooling of molecules to date.

To compute energies and decay strengths, we construct a numerical representation of the Hamiltonian, $H = H_{\text{rot}} + H_{\text{sr}}$, in a Hund’s case (b) symmetric top basis, $|N, K, S, J, M_J\rangle$, using matrix elements taken from the literature [158,161]. Numerical diagonalization yields eigenenergies and eigenstates with a block structure, as expected, on the basis of symmetry consideration [162]. When $K_a$ ($K_c$) sets the dominant level structure, the $K_c$ ($K_a$) label distinguishes the two parity components with a given $J$.

Note that $H_{\text{rot}}$ contains matrix elements with $\Delta K = \pm 2$ selection rules, although these are still diagonal in $N$ and may only affect the low-$N$ states of interest to laser cooling through higher-order perturbations. Here, $H_{\text{rot}}$ includes both diagonal $(\langle NK|H_{\text{rot}}|NK\rangle)$ and off-diagonal $(\langle NK|H_{\text{rot}}|NK \pm 2\rangle)$ elements, and those with $\Delta K = \pm 1$ (which vanish for molecules with orthorhombic symmetry) scale with $(\epsilon_{ab} + \epsilon_{ba})$ [158]. As discussed in Appendix C, both of these are expected to be negligible. The matrix elements that preserve $K$ but couple rotational states with $\Delta N = \pm 1$ scale directly with $\epsilon_{ab}$ and $\frac{1}{2}(\epsilon_{hb} + \epsilon_{hc})$. As will be discussed in Appendix C, these terms are not entirely negligible but likely affect photon scattering only after close to or greater than $10^5$ optical cycles.

The eigenstates are most conveniently described by the labels $N_{K_a,K_c}$, where $N$ is the rotational quantum number, and $K_a$ and $K_c$ are labels that correlate to the projection of $N$ onto the $a$ and $c$ axes in the prolate and oblate symmetric top limits, respectively. When we adopted the example of near-prolate ATMs in Fig. 6, the small moment of inertia along the $a$ axis means that levels of different $K_a$ values are widely spaced, each containing a ladder of $N$ rotational states. For each $N$, there are sublevels split by the spin-rotation interaction as the asymmetry doubling. In general, the spin-rotation splitting grows with increasing $N$, while for a given $N$, the asymmetry doubling shrinks rapidly with increasing $K_a$. It is straightforward to construct a similar diagram for ATMs with any other value of the Ray’s asymmetry parameter $\kappa \equiv (2B - A - C)/(A - C)$, although the relative energies will shift. For near-oblate species, the roles of $K_a$ and $K_c$ are swapped, but otherwise the conclusions remain the same.

**APPENDIX C: ROTATIONAL BRANCHES AND CLOSURE**

Electric dipole selection rules can be determined by symmetry considerations applied to the rovibronic states. In particular, electric dipole transitions must obey the constraint [163]

$$\Gamma'' \otimes \Gamma' \supset \Gamma^*,$$

where $\Gamma''$ and $\Gamma'$ are the symmetries of the rovibronic states involved in the transition, and $\Gamma^*$ is the electric dipole representation of the molecular symmetry group. General selection rules on the values of $K_a$ and $K_c$ can be derived by working in the rotation group $D_2$. It is useful to compare the rotational selection rules (see Table IV) to the case of symmetric top molecules. Consider first a prolate symmetric top molecule, e.g., CaCCH (see Fig. 4, column 2), in which the $a$ axis corresponds to the symmetry axis, and the $b$ and $c$ axes lie in a plane perpendicular to this axis. In this case, transitions can be characterized as “parallel” or “perpendicular” depending on whether the transition dipole moment is along the symmetry axis or perpendicular to it. Parallel transitions have the selection rule $\Delta K = 0$, while perpendicular transitions have the selection rule $\Delta K = \pm 1$. As the molecule distorts from symmetric, the equivalence between the $b$ and $c$ axes is broken. Nonetheless, for a near-prolate symmetric top, the $a$-type transition is still of approximate parallel nature, and the $a$-type selection rules in Table IV are justified. Similar reasoning provides an intuitive basis for selection rules in approximately perpendicular transitions, as well as for near-oblate tops.

ATMs with $C_{2v}$ symmetry are expected to follow these patterns rigorously, as the three components of $\hat{\mu}$ belong to a different irreducible representation of the symmetry group [163]. Similarly, the electronic orbitals are predominantly aligned along the principal axes. In the more general case of $C_i$ symmetry, a given excited state can decay by coupling to one of several components of $\hat{\mu}$; for example, both $\hat{\mu}_a$ and $\hat{\mu}_b$ belong to $A'$. Geometrically, the transition moments, determined by the electronic wave functions, and the principal axes, determined by the geometry of the molecule, are misaligned. This result means that either $K_a$ or $K_c$, or both, can change in a generic electronic decay. For example, in CaSH, the $\hat{C} \rightarrow \hat{X}$ transition can couple to either $\hat{\mu}_a$ or $\hat{\mu}_c$. In addition, off-diagonal couplings between electronic manifolds due to a term $(\epsilon_{ab} + \epsilon_{ba})$ in the spin-rotation Hamiltonian can mix the rotational state labels $K_a$ and $K_c$. The magnitude of this effect will vary from species to species depending on the spin-rotation tensor.

A simple estimate for the relative intensity of $c$-type transitions to that of $a$-type transitions in molecules with $C_i$ symmetry can be obtained by considering the relative alignment between the principal axes (which determine the rotational structure) and the metal-ligand bond (which determines the local symmetry of the valence orbitals). In the case of molecules like CaSH and SrSH, the angle between the metal-sulfur bond and the $a$ axis is typically about 0.5°. This angle implies that roughly 1 in $10^5$ decays from $\hat{C} \rightarrow \hat{X}$ couples to $\hat{\mu}_c$. This number is consistent with the transition dipole moments computed by our $ab \text{ initio}$ calculations in Table IV of the Supplemental Material [89].
which predict transition strengths that differ by a factor of about 100. Thus, if fewer than $10^4$ photon scatters are required, laser cooling of CaSH can proceed using only the scheme from a single panel of Fig. 6. To scatter additional photons, either microwave remixing or additional laser frequency components can be used to repump population that decays due to these small branches.

The effect of state mixing, Eq. (B3), is most prominent in the electronically excited states. For example, rotation about the $a$ axis can induce mixing between the $\tilde{A}$ and $\tilde{B}$ states, which will tend to unquench the orbital angular momentum. Reference [105] provides a physical intuitive picture: In the nonrotating molecule, the $\tilde{A}$ and $\tilde{B}$ states correspond to standing $p\pi$ orbitals—one along the $b$ axis and one along the $c$ axis—with quenched orbital angular momentum. As the molecule begins to rotate, the $p\pi$ orbitals follow adiabatically but eventually decouple from the nuclear framework as a net orbital angular momentum $\langle L_z \rangle$ is induced. This process is reflected, for example, in the values of $\epsilon_{aa}$ measured for the hydrosulfides and monoamides [59,105]. The magnitude of this interaction grows with increasing $K_a$ and in the limit of large $K_a$, $\epsilon_{aa} \rightarrow A_{SO}$. Similarly, rotation about the $b$ axis can lead to interactions between $\tilde{A}$ and $\tilde{C}$ states, and rotation about the $c$ axis can mix the $\tilde{B}$ and $\tilde{C}$ states [60].

Whether, and to what degree, these interactions affect laser cooling will differ from one species to another. Generally speaking, for near-prolate species, the interaction between $\tilde{A}$ and $\tilde{B}$ states is of little concern, as rotationally closed optical cycles with hybrid $bc$-type selection rules differ only by which subset of asymmetry doublet components are driven by the laser [see Figs. 6(b) and (c)]. The full set of levels typically covers less than 1 GHz and can easily be addressed with a single laser. Mixing between the $\tilde{C}$ state and either $\tilde{A}$ or $\tilde{B}$ introduces bands with selection rule $\Delta K_a = 0, \pm 1$, requiring the addition of one additional laser per vibrational band. Alternatively, microwave remixing can be used to return population to the optical cycle. This case is very similar to the case of symmetric top molecules [33]. For near-oblate molecules, the same ideas apply, though with the role of the $a$ and $c$ axes reversed. Finally, as the orbital angular momentum of the $\tilde{A}$ or $\tilde{C}$ state is unquenched ($\langle L_z \rangle \rightarrow 1$), the level structure will approach that of a $^3I_1$ state in symmetric species. In such a case, an additional rotational decay to the $2_{11}, J' = 3/2$ level may increase in strength. This case is analogous to the decay to the $N = 2, K = 1, J = 3/2$ parity doublet in symmetric top species, which is easily remixed into the optical cycle [33].

Interactions among the electronically excited states may unquench the electronic orbital angular momentum and induce transitions with $\Delta N = \pm 2$. The relative strength of such decays will vary from species to species, and for excited states with spin-rotation constants $\epsilon_{aa}$ that are small relative to the spacing between vibronic state levels (as is common), the probability of these decays will approach zero. For instance, $\epsilon_{aa}'/\Delta E_{B-A} \sim 10^{-2}$ in the molecules CaSH and SrSH [53]. Thus, this mixing will likely be relevant once more than about $3 \times 10^4$ photons are scattered. Note, too, that the $\Delta N = \pm 2$ rovibronic decay will be induced as the $\tilde{A}$ and $\tilde{B}$ states coalesce into the $\tilde{A}^2E$ state in molecules with near-$C_{3v}$ symmetry. This decay requires the addition of a laser frequency (or microwave) to repump the $2_{1K_a} J'' = 3/2$ state, as discussed in Ref. [33]. Note that this state is the same one that requires remixing for reasons already discussed above; i.e., this is not a new loss channel.

Off-diagonal terms of the spin-rotation interaction within the ground state may also be relevant. Note that $H_{sa}$ may mix states that differ by $\Delta K_a = \pm 2$ and by $\Delta N = \pm 1$. For the ATMs proposed here, the $\Delta K = \pm 2$ terms are proportional to $(\epsilon_{bb}^c - \epsilon_{cc}^c) \approx 10^{-4}$ cm$^{-1}$ [53]. Compared to the typical spacing between states differing in $K_a$ by $\pm 2$ ($\approx$10–100 cm$^{-1}$), such terms lead to rotational losses at a level close to or less than $10^{-8}$ and are negligible. Similarly, for nonorthorhombic molecules, there will be matrix elements of $H_{sa}$ that can mix states with $\Delta K = \pm 1$. Such matrix elements are proportional to $(\epsilon_{ab}^c + \epsilon_{ba}^c)/2$ [158], which is typically small ($\approx$10$^{-4}$ cm$^{-1}$ in CaSH [53]) compared to the spacing between connected levels. Again, we expect this case to lead to mixings at a level close to $10^{-8}$, which is largely negligible. The $\Delta N = \pm 1$ terms in $H_{sa}$ are potentially more relevant. Mixing of the $2_{1K_a} J_c$ rotational level into the $1_{K_a} J_c$ ground state is most likely to affect the rotationally closed optical cycles. At first order in perturbation theory, the admixed amplitude will be $\zeta \sim \epsilon_{aa}/4B_a$, where $a = a, b, c$ labels the principal axes, and typical values of $\zeta \approx 1$–5 × 10$^{-3}$ [56,164]. Such mixing will present a relevant loss channel after scattering close to or greater than $5 \times 10^4$ photons, which is more than required for typical laser cooling. If such losses begin to affect the optical cycle, remixing via a single additional laser frequency or microwave transitions can be used to increase the photon budget. Once again, this case is the same $N'' = 2$ state that may require repumping for other reasons, so these laser frequencies may already be present.


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